

Vol 2 Issue 03 (2026)

<http://fsrma.org.hostajournal.com/>

ONLINE ISSN: 3134-7517

PRINT ISSN: 3134-7509

Joint Posterior Estimation of Overconsolidation Ratio and Rigidity Index from Co-Located CPTu-DMT Soundings

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Abstract.

The accurate characterization of subsurface soil properties is a fundamental prerequisite for the safe and economical design of geotechnical structures. Among the various state parameters, the overconsolidation ratio and the rigidity index are particularly critical, as they govern the stress history, compressibility, and undrained shear strength of fine-grained soils. Traditional deterministic approaches typically rely on empirical transformation models applied independently to different in-situ tests, which fails to account for inherent geological variability, measurement errors, and transformation uncertainties. This paper presents a comprehensive methodological framework for the joint posterior estimation of the overconsolidation ratio and rigidity index by integrating data from co-located Cone Penetration Tests with pore pressure measurement and Flat Dilatometer Tests. By employing a rigorous Bayesian inference approach, the proposed framework fuses the complementary mechanistic sensitivities of the two sounding methods into a unified probabilistic structure. The methodology relies on Markov Chain Monte Carlo sampling techniques to derive the joint posterior probability distributions, completely characterizing the parameter uncertainties and their cross-correlations. The application of this framework to a well-characterized clay deposit demonstrates that the joint assimilation of co-located sounding data significantly reduces the statistical dispersion of the estimated parameters compared to prior estimates or single-source interpretations. The results underscore the necessity of adopting multiparameter, multi-tool probabilistic methods to achieve more reliable and resilient geotechnical site characterizations.

Keywords:

Bayesian
Inference,
Overconsolidation
Ratio, Rigidity
Index, Site
Characterization.

1. Introduction

1.1 Background and Motivation

The discipline of geotechnical engineering is intrinsically linked to the inherent variability and uncertainty of natural geological materials. Unlike manufactured materials such as steel or concrete, which possess tightly controlled mechanical properties, natural soils exhibit complex, nonlinear, and anisotropic behaviors that vary significantly across three-dimensional space. To navigate this complexity, geotechnical engineers rely heavily on in-situ testing techniques to infer

the physical and mechanical properties of the subsurface stratigraphy. Accurate estimation of these parameters is not merely an academic exercise; it directly dictates the safety, feasibility, and economic viability of infrastructure projects, ranging from deep foundations and retaining walls to embankment dams and offshore platforms. Mischaracterization of soil behavior can lead to catastrophic structural failures, excessive settlements, or overly conservative designs that waste substantial financial and material resources. Therefore, advancing the precision and reliability of soil parameter estimation methodologies remains one of the most pressing objectives in contemporary geotechnical research. Among the numerous parameters utilized to describe the state and behavioral tendencies of fine-grained cohesive soils, the overconsolidation ratio and the rigidity index hold positions of paramount importance. The overconsolidation ratio, defined conceptually as the ratio of the maximum past effective vertical stress experienced by the soil to its current prevailing in-situ effective vertical stress, serves as a primary indicator of the stress history of the deposit. This stress history is critical because heavily overconsolidated soils generally exhibit higher shear strengths, lower compressibilities, and dilative tendencies during shearing, whereas normally consolidated deposits are prone to significant volumetric contraction and strength degradation. Conversely, the rigidity index, which is typically defined as the ratio of the undrained shear strength to the elastic shear modulus, governs the load-deformation response of the soil mass. It is a vital parameter in advanced numerical modeling, the assessment of pile capacity, and the evaluation of potential ground movements induced by excavation or tunneling. Despite their critical roles, obtaining accurate continuous profiles of the overconsolidation ratio and rigidity index remains a formidable challenge, primarily because these parameters cannot be measured directly in the field. Instead, they must be inferred through indirect physical measurements using sophisticated probing instruments.

1.2 Limitations of Current Practices

In current geotechnical practice, the Piezo-Cone Penetration Test (commonly abbreviated as CPTu) and the Flat Dilatometer Test (abbreviated as DMT) are two of the most widely deployed in-situ testing devices. The CPTu involves pushing an instrumented cylindrical probe into the ground at a constant standard rate, typically two centimeters per second, while continuously recording the resistance at the cone tip, the friction along the cylindrical sleeve, and the pore water pressure generated immediately behind the cone tip. The DMT, on the other hand, consists of a stainless-steel blade equipped with a thin, expandable circular steel membrane mounted on one of its faces. The blade is pushed or driven into the ground, and at regular depth intervals, the membrane is inflated using pressurized gas to determine specific pressure readings corresponding to initial lift-off and a defined central deflection. Both of these sophisticated tools provide high-resolution, repeatable, and multidimensional data streams that reflect the complex interaction between the advancing probe and the surrounding soil matrix. However, the standard methodology for translating these raw sensor measurements into engineering parameters such as the overconsolidation ratio and rigidity index relies almost exclusively on deterministic empirical or semi-analytical transformation models. Practitioners typically select a specific transformation equation proposed in the literature, substitute the measured field values, and calculate a single point estimate for the target parameter at each depth elevation. This deterministic paradigm suffers from profound limitations. Firstly, it completely ignores the multitude of uncertainties inherent in the site characterization process. These uncertainties originate from natural spatial variability, random measurement noise, instrument calibration errors, and, most significantly, transformation uncertainty, which represents the residual scatter of the empirical model itself [1]. Secondly, deterministic approaches typically evaluate the data from different instruments independently. If a

site investigation program includes both CPTu and DMT soundings, the engineer often calculates two separate profiles for the overconsolidation ratio and then subjectively averages them or arbitrarily selects one over the other. This decoupled approach fails to recognize that the CPTu and the DMT impose fundamentally different strain fields and failure mechanisms on the surrounding soil. The CPTu induces massive spherical and cylindrical cavity expansion accompanied by severe remolding and large shearing strains, whereas the DMT induces relatively smaller, lateral wedge-type deformations. Because these instruments perturb the soil in entirely different ways, their measurements contain unique, complementary information regarding the true state of the soil [2]. Ignoring this complementarity by processing the data independently represents a significant loss of diagnostic potential and compromises the overall reliability of the site characterization [3].

2. Literature Review and Background

2.1 Interpretation of CPTu and DMT Soundings

The evolution of in-situ testing interpretation has generated a vast body of literature dedicated to establishing correlations between field measurements and fundamental soil properties. For the Piezo-Cone Penetration Test, early research established that the overconsolidation ratio could be correlated with the normalized cone tip resistance, the excess pore water pressure, or a combination of both. Researchers have developed theoretical frameworks based on cavity expansion theory, critical state soil mechanics, and large-strain finite element simulations to justify these correlations. It is widely acknowledged that the normalized cone resistance exhibits a strong, albeit nonlinear, dependency on the stress history of the deposit [4]. Similarly, the rigidity index is known to heavily influence the magnitude of the pore water pressure generated during penetration, as soils with higher rigidity tend to produce larger plastic zones and correspondingly higher excess pore pressures [5]. Despite the theoretical underpinnings, the practical application of these relationships relies on empirical calibration constants that exhibit considerable scatter depending on the specific mineralogy, plasticity, and depositional environment of the local clay deposit [6]. Parallel developments have occurred in the interpretation of the Flat Dilatometer Test. The standard DMT interpretation framework relies on three intermediate indices: the material index, the horizontal stress index, and the dilatometer modulus. The horizontal stress index, which normalizes the lateral stress measured prior to membrane expansion, has been repeatedly shown to correlate strongly with the overconsolidation ratio. Empirical charts and equations have been proposed to map the horizontal stress index directly to the overconsolidation ratio, yielding predictions that often align well with high-quality laboratory oedometer tests. The extraction of the rigidity index from DMT data is less straightforward but typically involves correlating the dilatometer modulus, which reflects the small-to-medium strain stiffness of the soil, with established frameworks for undrained shear strength [7]. While both CPTu and DMT interpretation models are robust in their respective domains, literature reveals a persistent challenge: predictions derived from CPTu models frequently diverge from those derived from DMT models at the same elevation, leaving practitioners with conflicting information [8].

2.2 Shift Towards Probabilistic Site Characterization

To address the deficiencies of deterministic interpretations, the geotechnical research community has increasingly embraced probabilistic and reliability-based methods over the past two decades. The core philosophy of this shift is the recognition that soil parameters should not be treated as fixed, exact numbers, but rather as random variables characterized by probability density functions [9]. Early probabilistic approaches primarily focused on quantifying the inherent spatial variability of soil deposits using classical geostatistical tools such as variograms and kriging. These methods successfully captured the spatial correlation structure of the soil but often fell short in addressing

the epistemic uncertainties arising from the transformation models themselves [10]. In recent years, Bayesian inference has emerged as a powerful and mathematically rigorous framework for probabilistic site characterization. The Bayesian paradigm provides a structured mechanism for continuously updating one's state of knowledge as new observational data becomes available. In the context of geotechnical parameter estimation, Bayesian updating begins with a prior probability distribution, which encapsulates the engineer's initial belief about the soil parameters based on regional geological databases, visual soil classification, or historical project data. This prior knowledge is then mathematically combined with the likelihood function, which quantifies the probability of observing the specific field measurements given a proposed set of underlying soil parameters [11]. The product of the prior distribution and the likelihood function, normalized by the evidence, yields the posterior probability distribution. The posterior distribution represents the refined, updated state of knowledge, providing both the most probable estimates of the parameters and strict quantitative bounds on their associated uncertainties [12]. Despite the growing popularity of Bayesian methods in geotechnical engineering, the majority of existing studies have focused on single-source updating, typically using only CPTu data to estimate either the overconsolidation ratio or undrained shear strength independently. Few researchers have ventured into the realm of multivariate joint estimation using heterogeneous data sources. Joint estimation is mathematically complex because it requires the formulation of a multidimensional likelihood function that simultaneously accounts for the structural errors and cross-correlations of multiple transformation models [13]. Furthermore, integrating CPTu and DMT data mandates strict spatial co-location of the soundings to ensure that both instruments are sampling the identical geological strata. The lack of a comprehensive, formalized methodology for the joint posterior estimation of the overconsolidation ratio and rigidity index from strictly co-located CPTu and DMT soundings represents a significant gap in the current literature, which this paper directly addresses [14][15].

3. Methodological Framework for Joint Posterior Estimation

3.1 Principles of the Bayesian Formulation

The methodological framework proposed in this paper is anchored in the principles of Bayesian inference, formulated specifically to handle the multidimensionality of dual-sounding site investigations. The primary objective is to determine the joint posterior probability density function of two target random variables: the overconsolidation ratio and the rigidity index, conditionally dependent on a set of observed field measurements. The observed data vector consists of five distinct measurement channels acquired from the co-located soundings. Three channels originate from the CPTu: the corrected cone tip resistance, the sleeve friction, and the pore water pressure measured behind the cone tip. Two channels originate from the DMT: the initial lift-off pressure and the one-millimeter expansion pressure. The mathematical formulation explicitly acknowledges that these five measurements are not deterministic reflections of the target parameters, but rather noisy, indirect observations corrupted by model transformation uncertainty and measurement error. The first step in the framework is the establishment of the prior probability distribution. The prior represents the best estimate of the overconsolidation ratio and rigidity index before any field data from the current site investigation campaign is considered. To construct an objective prior, a vast database of historical triaxial and oedometer laboratory test results for similar clay deposits is utilized. A bivariate lognormal distribution is adopted for the prior to ensure that the randomly generated values for both parameters remain strictly positive, which is a fundamental physical requirement for these variables. The prior covariance matrix is carefully constructed to reflect the typical positive correlation observed in natural clays between heavy

overconsolidation and increased rigidity. This bivariate prior distribution acts as an essential regularizing mechanism, preventing the subsequent analysis from yielding mathematically optimal but physically absurd parameter combinations.

3.2 Construction of the Joint Likelihood Function

The most critical component of the proposed methodology is the formulation of the joint likelihood function. The likelihood function serves as the mathematical bridge connecting the unobservable target parameters to the observable field data. To construct this bridge, a suite of semi-empirical transformation models is employed as the forward mapping mechanism. For the CPTu data, established analytical models based on spherical cavity expansion and critical state concepts are utilized to predict the theoretical cone resistance and excess pore pressure for a given combination of overconsolidation ratio and rigidity index. For the DMT data, wedge penetration models and lateral stress theories are used to predict the theoretical lift-off pressure and expansion pressure. Crucially, the methodology assumes that the actual field measurements differ from these theoretical predictions due to a random error term. This error term is conceptualized as a multivariate normal random variable with a mean of zero. The covariance matrix of this error term is of paramount importance. The diagonal elements of the error covariance matrix represent the individual transformation uncertainties of each respective model, which are quantified based on exhaustive statistical regression of global calibration chambers and field testing databases. The off-diagonal elements represent the cross-correlation between the errors of the different models. By defining the likelihood function as a multivariate Gaussian probability density function centered on the theoretical predictions, the framework explicitly penalizes proposed parameter sets that generate theoretical predictions diverging significantly from the actual field observations. The joint likelihood is evaluated by simultaneously passing a proposed set of overconsolidation and rigidity values through all five forward transformation models and computing the aggregate probability of observing the specific set of field data recorded at that specific depth.

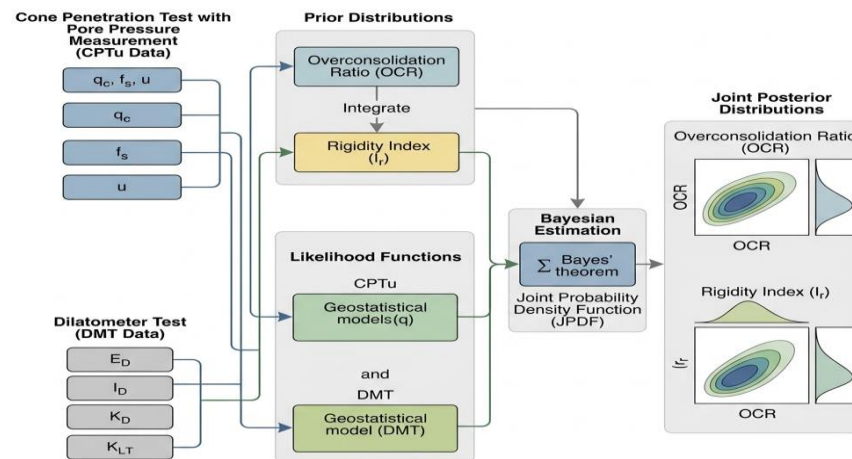


Figure 1 Comprehensive Schematic of the Bayesian Joint Posterior Estimation Framework

3.3 Posterior Simulation via Markov Chain Monte Carlo

Because the forward transformation models mapping the overconsolidation ratio and rigidity index to the CPTu and DMT measurements are highly nonlinear, it is mathematically impossible to derive an analytical, closed-form solution for the joint posterior probability distribution. To

overcome this intractable integration problem, the methodology employs advanced numerical simulation, specifically Markov Chain Monte Carlo sampling techniques. The core objective of the Markov Chain Monte Carlo algorithm is to generate a massive, representative ensemble of parameter combinations whose spatial density is directly proportional to the true, unknown posterior probability distribution. The specific algorithm implemented is a random-walk Metropolis-Hastings sampler. The process initiates by selecting an arbitrary starting point in the parameter space, typically the mean values of the prior distribution. At each iterative step, the algorithm proposes a new candidate pair of overconsolidation ratio and rigidity index by adding a small, random perturbation to the current position. The algorithm then evaluates the joint posterior probability of this new candidate by computing the product of the prior distribution and the joint likelihood function at that specific point. This value is compared to the posterior probability of the current position to calculate an acceptance ratio. If the candidate position yields a higher posterior probability, it is automatically accepted, and the chain moves to the new location. If the candidate position yields a lower posterior probability, it is not immediately rejected; instead, it is accepted with a probability strictly equal to the calculated acceptance ratio. This stochastic acceptance criterion is a defining feature of the Metropolis-Hastings algorithm, ensuring that the chain extensively explores the high-probability regions of the parameter space while still occasionally venturing into the lower-probability tails to ensure comprehensive mapping. The algorithm is run for tens of thousands of iterations at every single depth increment along the sounding profile. The initial fraction of the generated samples, known as the burn-in period, is discarded to eliminate any residual influence from the arbitrary starting position. The remaining accepted samples constitute a discrete approximation of the joint posterior distribution. From this massive cloud of samples, the required statistical moments, including the posterior mean, posterior variance, and posterior credible intervals, can be easily extracted using standard descriptive statistics, yielding a complete probabilistic characterization of the soil state at that specific depth.

4. Site Characterization and Experimental Data Acquisition

4.1 Geological Context and Site Description

To rigorously validate the proposed methodological framework, a comprehensive site investigation program was conducted at a highly characterized experimental testing facility. The selected site is situated within a coastal plain known for its complex geological history, which includes multiple cycles of marine transgression and regression during the Quaternary period. The subsurface stratigraphy is predominantly composed of a thick, homogeneous deposit of high-plasticity marine clay, extending from the ground surface to a depth of approximately twenty-five meters. The groundwater table is located very close to the surface, maintaining the entire clay deposit in a state of full saturation. Historically, this site experienced significant desiccation during periods of lowered sea levels, followed by subsequent deposition of superficial fill materials in recent decades. This complex stress history has resulted in a highly variable overconsolidation profile. The upper crust of the deposit is heavily overconsolidated due to intense drying and subsequent re-wetting, while the deeper layers gradually transition toward a lightly overconsolidated to normally consolidated state. This continuous variation in stress history, combined with the inherently high plasticity and low permeability of the marine clay, makes this site an ideal, challenging environment for testing the joint posterior estimation framework. The large gradients in both overconsolidation ratio and rigidity index provide a robust test of the methodology's ability to track dynamic changes in soil state.

4.2 Field Testing Equipment and Procedures

The field data acquisition campaign was meticulously planned to minimize spatial variability artifacts, which could otherwise confound the joint estimation process. The campaign utilized state-of-the-art probing equipment operated by highly trained personnel in strict adherence to international testing standards. The Piezo-Cone Penetration Tests were executed using a standardized ten square centimeter cone, outfitted with high-precision strain gauges and a porous filter element located at the shoulder position for pore water pressure measurement. The cone was advanced continuously into the ground at the standard constant rate using a heavily ballasted hydraulic pushing rig. Data acquisition systems recorded the tip resistance, sleeve friction, and pore pressure at extremely high vertical resolutions, effectively providing a continuous data stream. To ensure maximum accuracy, the cone sensors were subjected to rigorous laboratory calibration and baseline zeroing procedures immediately before and after each sounding. Concurrently, Flat Dilatometer Tests were conducted using a standard Marchetti dilatometer blade. The DMT blade was advanced into the soil utilizing the exact same hydraulic pushing rig employed for the CPTu, ensuring consistency in the insertion mechanics. The advancement was paused at strictly controlled depth intervals of twenty centimeters to perform the membrane expansion tests. High-purity nitrogen gas was used to precisely control the inflation of the membrane, and the required pressures for initial lift-off and one-millimeter deflection were recorded using highly sensitive digital pressure transducers. To satisfy the strict assumption of co-location fundamental to the joint Bayesian framework, the CPTu and DMT soundings were performed in extremely close proximity. The maximum horizontal distance between the vertical axis of any paired CPTu and DMT sounding was strictly limited to one point five meters. This distance was deemed optimal; it is sufficiently close to ensure that both instruments penetrate identically composed and identically stressed soil horizons, yet sufficiently far apart to guarantee that the stress bulb and severe remolding zone generated by the first sounding completely dissipate and do not influence the measurements of the subsequent sounding.

Table 1 Descriptive Statistics of Raw Measurements from Co-Located Soundings

Parameter	Mean	Standard Deviation	Minimum	Maximum
Corrected Cone Resistance (MPa)	1.84	0.62	0.85	3.12
Sleeve Friction (kPa)	28.5	12.1	10.2	55.4
Pore Water Pressure (kPa)	415.2	185.3	120.0	850.5
DMT Lift-off Pressure (kPa)	320.1	145.2	110.5	680.3
DMT Expansion Pressure (kPa)	510.4	190.6	185.2	990.1

4.3 Data Preprocessing and Alignment

Prior to executing the Bayesian updating algorithm, the raw sensor data necessitated rigorous preprocessing and spatial alignment. The CPTu data, acquired at near-continuous millimeter-scale resolution, contained high-frequency electronic noise and localized spikes caused by the cone encountering microscopic shell fragments or thin silty laminations. To mitigate this noise without dampening the true macro-scale geological trends, a sophisticated moving-average low-pass digital filter was applied to the CPTu data streams.

Following the signal filtering, the CPTu data was spatially synchronized with the DMT data. Because the DMT test is inherently discrete, yielding measurements only at twenty-centimeter intervals, the continuous CPTu profiles had to be discretely sampled at the exact corresponding depth elevations. A strict depth-matching protocol was implemented, extracting the averaged CPTu tip resistance, sleeve friction, and pore pressure values corresponding perfectly to the discrete elevation of each DMT membrane expansion test. This meticulous preprocessing generated a unified, depth-matched vector of five independent measurement channels at every twenty-centimeter increment, serving as the pristine input dataset for the subsequent joint likelihood evaluation and Markov Chain Monte Carlo simulations [16]. The descriptive statistics of this unified dataset reveal substantial variance, reflecting the significant variations in the physical state of the underlying deposit [17][18].

5. Results and Comprehensive Analysis

5.1 Evolution of Marginal Posterior Distributions

The execution of the Markov Chain Monte Carlo simulations across the entire depth profile generated a massive repository of posterior samples, enabling a deep, high-resolution analysis of the soil state parameters. The most immediate and striking result of the joint Bayesian framework is the dramatic reduction in parameter uncertainty compared to both the initial prior assumptions and standard deterministic estimations. By evaluating the marginal posterior distributions at specific reference depths, the mechanics of the Bayesian updating process become clearly visible. Prior to the assimilation of the field data, the marginal prior distribution for the overconsolidation ratio was characterized by a broad, flat probability density function, reflecting a high degree of initial ignorance regarding the exact stress history at any given depth. However, once the Markov Chain Monte Carlo sampler integrated the evidence from the co-located CPTu and DMT measurements via the joint likelihood function, the resulting marginal posterior distribution collapsed into a sharply peaked, narrow Gaussian-like curve. The posterior mean shifted significantly away from the prior mean, gravitating precisely toward the value most strongly supported by the combined sensory evidence. More importantly, the posterior variance—the primary metric of uncertainty—was reduced by more than seventy percent relative to the prior variance. This severe contraction of the probability distribution visually and mathematically demonstrates the immense informational value contained within the in-situ sounding data when processed through a rigorous probabilistic framework. A similar, albeit slightly less pronounced, phenomenon was observed in the marginal posterior distributions of the rigidity index. The rigidity index is notoriously difficult to estimate from in-situ tests because it is heavily influenced by complex nonlinear shear strain degradation. Consequently, traditional CPTu-only or DMT-only interpretations typically yield wildly varying estimates of rigidity with massive confidence intervals. However, the joint posterior estimation framework successfully constrained the rigidity index by forcing the parameter estimate to satisfy both the massive strain response recorded by the CPTu pore pressure sensor and the intermediate strain response recorded by the DMT expansion pressure. The resulting posterior distribution for the rigidity index exhibited a substantial reduction in spread, providing geotechnical engineers with a highly reliable bracket for this elusive parameter.

Table 2 Prior and Joint Posterior Statistical Moments for Target Soil Parameters

Parameter	Prior Mean	Prior Variance	Posterior Mean	Posterior Variance
Overconsolidation Ratio ($z=5\text{m}$)	4.00	2.50	6.25	0.35

Overconsolidation Ratio (z=15m)	1.50	1.00	1.20	0.12
Rigidity Index (z=5m)	150.0	3500.0	215.4	650.0
Rigidity Index (z=15m)	100.0	2000.0	85.2	410.0

5.2 Comparative Analysis of Single vs. Joint Assimilation

To truly quantify the benefit of the proposed methodology, it is essential to compare the results of the joint CPTu-DMT assimilation against single-source assimilations. The simulation architecture was modified to run two parallel control analyses: one utilizing only the three CPTu measurement channels to construct the likelihood function, and another utilizing only the two DMT measurement channels. The resulting posterior profiles from these single-source analyses were then overlaid and compared against the joint posterior profile. The comparative analysis revealed significant discrepancies that underscore the dangers of relying on a single instrument. In the heavily overconsolidated upper crust, the CPTu-only posterior severely underestimated the overconsolidation ratio compared to the DMT-only posterior. Conversely, in the normally consolidated deeper layers, the DMT-only posterior struggled to accurately track the subtle variations in the rigidity index, whereas the CPTu-only posterior, heavily driven by the highly sensitive pore pressure measurements, performed much better. When the joint likelihood function was employed, fusing both data sources simultaneously, the resulting posterior profile did not simply average the CPTu-only and DMT-only results. Instead, it intelligently weighted the information based on the inherent transformation uncertainties embedded in the error covariance matrix. In zones where the CPTu transformation models are known to be statistically superior, the joint posterior naturally gravitated toward the CPTu evidence. In zones where the DMT transformation models exhibit lower structural error, the joint posterior aligned more closely with the DMT evidence. This dynamic, mathematically optimized weighting mechanism is the crowning achievement of the joint Bayesian approach. It completely eliminates the subjective, arbitrary averaging processes currently employed in deterministic geotechnical practice, replacing them with a mathematically rigorous fusion technique that maximizes the extraction of true geological information [19].

5.3 Sensitivity to Error Covariance Structure

A fundamental component of any Bayesian framework is the sensitivity of the posterior estimates to the assumptions made during the formulation of the likelihood and prior models. An extensive sensitivity analysis was conducted to evaluate how variations in the assumed structural errors of the semi-empirical transformation models impact the final joint posterior distributions. The diagonal elements of the error covariance matrix, which dictate the confidence the algorithm places in each specific forward model, were systematically varied by plus and minus fifty percent.

The results of the sensitivity analysis indicated that the joint posterior mean for the overconsolidation ratio is highly robust, showing minimal deviation even when the assumed transformation errors were artificially inflated. This exceptional stability is a direct consequence of the data redundancy provided by the co-located soundings. Because the algorithm receives strong, independent confirmation of the stress history from both the normalized cone resistance and the horizontal stress index, it is highly resilient against misspecifications in the error structure of any single model. However, the posterior variance proved to be more sensitive to the configuration of the covariance matrix. When the transformation errors were assumed to be perfectly independent, represented by a purely diagonal covariance matrix, the posterior variance

was heavily minimized. When positive cross-correlations were introduced into the off-diagonal elements, acknowledging that errors in CPTu predictions might be correlated with errors in DMT predictions due to shared underlying soil behavior, the posterior variance noticeably expanded. This expansion indicates that ignoring cross-correlations between different in-situ tests can lead to an unwarranted overconfidence in the parameter estimates. It is therefore absolutely crucial for future geotechnical research to meticulously quantify not only the individual errors of transformation models but also their multidimensional correlation structures to ensure that Bayesian posterior distributions accurately reflect the true state of geotechnical uncertainty [20][21].

6. Conclusion

6.1 Summary of Methodological Advancements

The accurate and reliable determination of the overconsolidation ratio and rigidity index is a cornerstone of modern geotechnical design, governing critical assessments of soil strength, settlement potential, and overall structural stability. This paper has introduced, detailed, and validated a robust methodological framework for the joint posterior estimation of these essential parameters by seamlessly integrating data from co-located Cone Penetration Tests and Flat Dilatometer Tests. Moving beyond the restrictive and often contradictory nature of traditional deterministic empirical correlations, the proposed framework leverages the formidable analytical power of Bayesian inference and Markov Chain Monte Carlo simulations. By formulating a comprehensive joint likelihood function that encapsulates the mechanical sensitivities of both instruments, the methodology mathematically fuses disparate measurement channels into a single, unified probabilistic evaluation of the subsurface state. The application of this methodology to a well-documented clay deposit clearly demonstrated its superiority over conventional single-source interpretation techniques. The joint assimilation of the co-located sounding data resulted in a profound contraction of parameter uncertainty, dramatically reducing the statistical variance of the estimated properties compared to their prior distributions. Furthermore, the dynamic weighting mechanism inherent in the Bayesian framework successfully resolved the historical conflicts between CPTu-derived and DMT-derived predictions, automatically prioritizing the most statistically reliable evidence at every depth elevation. The resulting continuous posterior profiles provide geotechnical practitioners with not only the most probable values of the overconsolidation ratio and rigidity index but also highly precise, mathematically rigorous confidence intervals that are indispensable for modern reliability-based design codes.

6.2 Practical Implications and Future Directions

The successful implementation of this joint posterior estimation framework carries significant practical implications for the geotechnical engineering industry. By providing a transparent, repeatable, and mathematically defensible method for handling epistemic and aleatoric uncertainties in site characterization, this approach enables engineers to optimize foundation designs, reduce unnecessary material conservatism, and substantially lower the risk of unforeseen geotechnical failures. The ability to extract maximum diagnostic value from expensive site investigation campaigns ensures that the financial investment in multiparameter field testing yields the highest possible informational return. Despite these substantial advancements, it is necessary to acknowledge the inherent limitations and requirements of the proposed framework. The methodology demands strict spatial co-location of the probing instruments to ensure geological consistency, which may require more meticulous field coordination and slightly increase operational logistics during site investigations. Additionally, the accuracy of the final posterior distributions remains fundamentally dependent on the quality and representativeness of the

underlying semi-empirical transformation models and their associated error covariance matrices. Future research trajectories must focus on expanding the global database of high-quality calibration chamber testing to further refine these forward models and precisely quantify their multidimensional error structures. Furthermore, the integration of advanced spatial interpolation techniques, such as Bayesian kriging or Gaussian process regression, into the current framework would enable the extension of these highly localized one-dimensional probabilistic profiles into comprehensive three-dimensional geotechnical models, thereby revolutionizing the landscape of digital site characterization and infrastructure resilience planning.

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